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2004 J. Phys. A: Math. Gen. 37 10727

(http://iopscience.iop.org/0305-4470/37/45/002)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **37** (2004) 10727–10734 PII: S0305-4470(04)83597-2

Universality classes of the three-dimensional *mn*-vector model

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Received 19 July 2004
Published 28 October 2004
Online at stacks.iop.org/JPhysA/37/10727
doi:10.1088/0305-4470/37/45/002

Abstract

We study the conditions under which the critical behaviour of the three-dimensional mn-vector model does not belong to the spherically symmetrical universality class. In the calculations, we rely on the field-theoretical renormalization group approach in different regularization schemes adjusted by resummation and extended analysis of the series for renormalization-group functions. We address the question why the renormalization-group perturbation theory expansions available for the model with a record (six loop) accuracy have not allowed so far for a definite answer about the universality class for certain particular values of dimensions m, n. We show that an analysis based on the marginal dimensions rather than on the stability exponents leads to the robust results about the phase diagram of the model.

PACS numbers: 05.50.+q, 64.60.Ak

According to the universality hypothesis [1], asymptotic properties of the critical behaviour remain unchanged for different physical systems if these are described by the same global parameters. The field-theoretical renormalization group (RG) approach [2] naturally takes into account the global parameters and derives properties of critical behaviour from long distance properties of effective field theories. In the present paper, we study the long-distance properties of the $d = 3$-dimensional mn-vector model which is introduced by the following effective field-theoretical Hamiltonian [3]:

$$\mathcal{H}[\phi(x)] = \int \mathrm{d}^d x \left\{ \frac{1}{2} \sum_{\alpha=1}^{n} [|\nabla \vec{\phi}^\alpha|^2 + \mu_0^2 |\vec{\phi}^\alpha|^2] + \frac{u_0}{4!} \sum_{\alpha=1}^{n} (|\vec{\phi}^\alpha|^2)^2 + \frac{v_0}{4!} \left(\sum_{\alpha=1}^{n} |\vec{\phi}^\alpha|^2 \right)^2 \right\}. \qquad (1)$$

Here, $\vec{\phi}^{\alpha} = (\phi^{\alpha,1}, \phi^{\alpha,2}, \ldots, \phi^{\alpha,m})$ is a tensor field of dimensions n and m along the first and the second indices, u_0 and v_0 are bare couplings and μ_0^2 is a bare mass-squared measuring the temperature distance to the critical point.

Depending on the choice of parameters m and n, the mn-vector model (1) is known to describe phase transitions of various microscopic nature. The choice $n = 1$ comprises a bunch of systems that are characterized by an O(m)-symmetric order parameter, while the limiting cases $n \to 0$ and $n \to \infty$ correspond to these systems exposed to the quenched [4] and annealed [5] disorder, respectively. The choice $m = 1$, arbitrary n, corresponds to the cubic model [6]. A separate interest is provided by cases $m = 2$, $n = 2$ describing [7] helical magnets and antiferromagnetic phase transitions in $TbAu_2$, DyC_2 as well as by cases $m = 2$, $n = 3$ describing antiferromagnetic phase transitions in TbD_2, Nd.

All of the mentioned cases of the mn-vector model were subjects of separate extensive studies (see e.g. [8–11] and references therein). They led to a consistent description of criticality in the O(m) and cubic systems. In particular, the precise estimates of the critical exponents of the cubic and of the random Ising model were established both within high-order expansions of the massive and minimal subtraction field-theoretical RG schemes [8, 11]. In contrast, the cases $m = 2$, $n = 2, 3$ remain controversial. The general non-perturbative consideration [6, 12] brings about that the theory (1) belongs to the O(2) universality class, while the perturbative field-theoretical RG approach yielded mixed data, neither proving nor rejecting this result [8, 13] despite analysing expansions available now within the record (six loop) accuracy [8]. However, the RG study of two-dimensional mn-vector model [14] reproduces all the non-perturbative results for the case $m, n \geqslant 2$.

The studies infer that an intrinsic feature of the theory (1) is an interplay between the O(k) ('trivial') universality class (with k of dimension m or mn) and a new universality class. In this paper, we address two problems that concern the crossovers in the mn-vector model and still attract attention. Firstly, we aim to obtain a map of universality classes of the theory (1) in the whole plane $m \geqslant 0$, $n \geqslant 0$. Such an analysis has been performed so far in the one-loop approximation [6]. We base the analysis on the high-loop expansions for the RG functions of the model (1) and its special cases; in order to refine the analysis we exploit Padé–Borel resummation [15, 16] of the (asymptotic) series under consideration. Secondly, we focus attention on cases $m = 2$, $n = 2$, 3 in order to explain why the highest orders of perturbation theory have not allowed so far to resolve what universality class is realized in the theory. We perform analysis in different perturbative schemes and show that only certain of them give a reliable answer.

We analyse the theory (1) applying the field-theoretical RG approach [2] within weak coupling expansion techniques. In the approach, a critical point corresponds to a reachable and stable fixed point (FP) of the RG transformation of a field theory. A FP $\{u^*, v^*\}$ is determined as a simultaneous zero of the β-functions describing the change of the renormalized couplings u and v under RG transformations and being calculated as perturbative series in renormalized couplings. The equations for the FP read

$$\begin{cases} \beta_u(u^*, v^*) = u^*\varphi(u^*, v^*) = 0, \\ \beta_v(u^*, v^*) = v^*\psi(u^*, v^*) = 0, \end{cases} \tag{2}$$

where we have explicitly shown that the structure of the β-functions allows their factorization for the effective Hamiltonian (1). We make use of both the dimensional regularization with minimal subtraction [17] and the fixed dimension renormalization at zero external momenta and non-zero mass (massive) [18] schemes. More precisely, we rely on the expansions for the β-functions that are known at $d = 3$ with the accuracy of six loops in the massive scheme [19] and with five-loop accuracy for the cases of O(m)-vector [20] and cubic models [21] in the minimal subtraction scheme.

For the analysis below it is important to mention that equations (2) for the FP can be solved technically in two complementary ways. A perturbative solution is obtained by an expansion of the FP coordinates in a small parameter ($\varepsilon = 4 - d$, with d being the space dimension of the model [22], in the minimal subtraction or massive schemes, or an auxiliary pseudo-ε parameter[5] in the massive scheme) around the Gaussian solution $\{u^* = 0, v^* = 0\}$. Such a method formally guarantees that the structure of solutions for the FPs remains the same after accounting for higher-order contributions once it has been established in the one-loop approximation. An alternative method (the 3d approach) consists in the solution of equation (2) numerically [18, 25] at a given order of perturbation theory and provides less control on a loopwise upgrade.

Within the perturbative approach, the conditions on m and n under which the critical behaviour of the mn-vector model (1) belongs to a non-trivial universality class are known as the Aharony conjecture and read [6]

$$n_c < mn < m_c n, \qquad n > 1. \tag{3}$$

Here, n_c and m_c stand for the marginal dimensions of the cubic model and of the random m-vector model. The conjecture is based on the one-loop stability analysis of four FP solutions compatible with equations (2). At $d < 4$, these are the Gaussian FP $\mathbf{G}$ $\{u^* = 0, v^* = 0\}$, the FPs $\mathbf{P}_{\mathrm{O(mn)}}$ $\{u^* = 0, v^* \neq 0\}$ and $\mathbf{P}_{\mathrm{O(m)}}$ $\{u^* \neq 0, v^* = 0\}$ describing theories with one ϕ^4 coupling and thus corresponding to the $\mathrm{O}(mn)$ and $\mathrm{O}(m)$ universality classes, and, finally, the mixed FP $\mathbf{M}$ $\{u^* \neq 0, v^* \neq 0\}$. It is the stability of the FP $\mathbf{M}$ that is necessary for the appearance of a new non-trivial critical behaviour.

Our 3d analysis of the theory (1) is obscured by the observation that at some choice of m and n more than four solutions for the FP are obtained. To convince ourselves that some of them are not a by-product of application of resummation procedures we propose to use the following argument. According to the basics of the RG theory, at the upper critical dimension $d = 4$, any ϕ^4 theory is governed by the Gaussian FP [2]. Therefore, any non-Gaussian solution at $d = 4$ is out of physical interest. If such a solution survives at any $d < 4$ and particularly at $d = 3$, we find it natural to consider it physically meaningless by continuity. The situation becomes less clear if a FP cannot be continually traced back to a certain solution at $d = 4$ because it disappears at some $3 < d_c < 4$. In this case the stability of the estimate for d_c as well as of the FP coordinates against application of different resummation procedures in different orders of perturbation theory might serve the purpose. We note here that the special case of the theory (1) with $m = 2$, $n = 2$ is known to have exact mapping onto the model describing non-collinear magnetic ordering [26]. Within the massive RG scheme, the standard six-loop 3d analysis of this model allowed one to find a stable FP which does not have the counterpart within the perturbative ε-expansion [27]. But one could not follow the evolution of the FP as d approaches 4 because in this case the resummation procedure was found to be ill-defined [28]. Recently, the problem was analysed within minimal subtraction scheme [29], but again the authors of [29] were not able to resum reliably the perturbative series for space dimensions up to $d = 4$.

To establish the map of universality classes of the theory (1) we use both perturbative and 3d analysis complementarily. We find that, in addition to the conditions (3), the *high*-order map is controlled by a degeneracy condition of *one*-loop equations for the FP [30]:

$$n = \frac{16(m-1)}{m(m+8)}. \tag{4}$$

[5] The pseudo-ε expansion technique was introduced by B G Nickel (unpublished); see reference [19] in [23]. For an application of the pseudo-ε expansion in models with several couplings see [10, 24].

Unlike order-dependent estimates for the marginal dimensions m_c and n_c, this equation is independent of the order of perturbation theory and is exact. We also observe that the results obtained with the account of high-order contributions differ qualitatively from those obtained in the one-loop approximation. We consider worth to mention three peculiarities.

(i) We find a domain in the mn-plane where the high-loop resummed β-functions produce no solution for the FP while such a solution exists in the one-loop approximation. In the mn-plane, the domain spans from the vicinity of the point $\{m = m_c, n = n_c/m_c\}$ upwards. There, we can solve equations (2) for the mixed FP reliably neither numerically at the fixed space dimension $d = 3$ nor by application of the pseudo-ε expansion. In particular, although the pseudo-ε expansion can be formally constructed there, its analysis by means of the Padé [15] or Padé–Borel–Leroy [16] technique produces highly chaotic values both for mixed FP coordinates and its stability exponents. (ii) We find a domain in the mn-plane where the 3d analysis reveals two solutions for the mixed FP **M** co-existing in opposite quadrants of the uv-plane. In the mn-plane, the domain is located below the point $\{m = m_c, n = n_c/m_c\}$. Yet, we are always able to establish that one of the two solutions is unphysical in the sense explained above. The described phenomenon is quite stable with respect to the order of perturbation theory and to the type of the resummation procedure applied. In the perturbative approach only one solution for the mixed FP is present. (iii) We observe that a smooth change of parameters m, n in some regions of the mn-plane can show up as a complex abrupt trajectory of the FP **M** in the uv-plane.

Realization of various universality classes of the theory (1) besides universal equations (3) and (4) depends on non-universal initial conditions for couplings. Certain physical interpretations of the mn-vector model (1) impose restrictions for the signs of the couplings. Namely, a group including the cubic model ($m = 1, \forall n$) and cases $m = 2$, $n = 2, 3$ imply u_0 of any sign and $v_0 > 0$ [6, 7], whereas the microscopic base of the weakly diluted quenched m-vector model strictly defines $u_0 > 0$, $v_0 \leqslant 0$ [4]. Taking into account such a division along with the pseudo-ε expansion-based estimates[6] for the marginal dimensions $n_c = 2.862 \pm 0.005$ [10] and $m_c = 1.912 \pm 0.004$ [32], we arrive at the high-loop map of the universality classes of the theory (1) as shown in figure 1. There, the domains governed by different universality classes are bounded by lines for marginal dimensions and the degeneracy line. The FP **M** is stable for values m and n contained in the dark regions. The stability regions of the FPs $\mathbf{P}_{\mathrm{O}(m)}$ and $\mathbf{P}_{\mathrm{O}(mn)}$ are horizontally and vertically hatched respectively. In the cross-hatched region in figure 1(*a*) both O(m) and $O(mn)$ FPs are stable. Here, the choice of the universality class depends on the initial values of couplings u, v. They can be located in one of the two domains of uv-plane created by the separatrix, which is determined by the unstable mixed FP. The blank region in figure 1(*b*) denotes the region of runaway solutions. Let us note that runaway solutions exist for the cubic-like models (figure 1(*a*)) also; however, there still exist regions of initial couplings u, v starting from which the stable FP is attained.

As we mentioned above, the high-loop analysis of the theory (1) encounters difficulties in some domains of the mn-plane. In particular, these are the domains where the mixed FP either disappears or can be given by two (physical and unphysical) solutions. Our direct calculations show that such domains (mainly) inset the regions where the FP **M** is expected to be unstable according to equations (3) and (4) and thus does not influence the analysis of figure 1. However, even if the solution for the FP **M** is steadily recovered, its stability analysis is obscured for some values of m, n. In particular, the latter is observed for the physically interesting cases $m = 2$, $n = 2$, 3. In the rest of the paper, we aim to show that the reliability of the stability analysis depends on the choice of a series that is assumed as its basis.

[6] These estimates are in agreement with those obtained by the other methods; see e.g. [31] and a review [8].

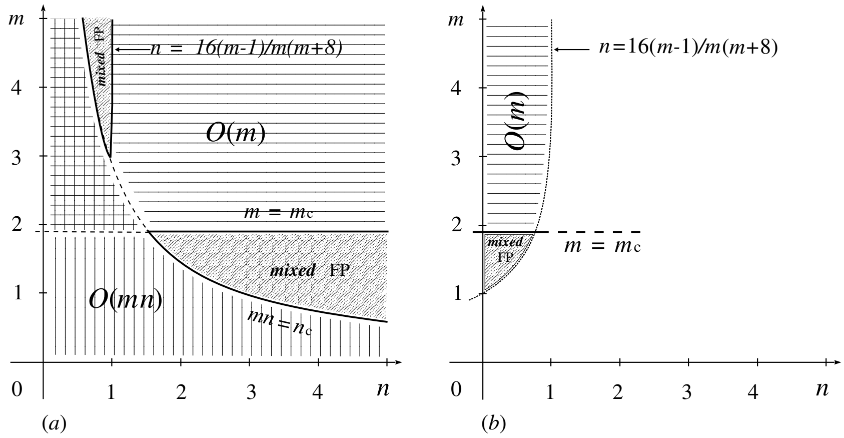

Figure 1. The domains of FP stability for the *mn*-vector model with different signs of couplings: $\forall\, u, v \geqslant 0$ (*a*) and $u > 0, v \leqslant 0$ (*b*). The mixed FP is stable for values *m* and *n* from dark regions. The stability regions of O(m) and O(mn) FPs are shaded by horizontal and vertical lines, respectively. Cross-hatched region (*a*) means that both O(m) and O(mn) FPs are stable (see text for details).

Indeed, the stability of a FP is governed by the condition $\Re(\omega_i) > 0$ with the stability exponents ω_i being the eigenvalues of the matrix of derivatives $B_{ij} = \partial\beta_{u_i}/\partial u_j$ ($u_i = u, v$) taken at the FP. For the case under consideration, $m = 2$, $n = 2$, 3, one of the eigenvalues (ω_2) is large and positive both at the FPs $\mathbf{P}_{\mathrm{O}(m)}$ and $\mathbf{M}$, so it is the sign of ω_1 that controls the stability of a FP. The non-peturbative considerations [6, 12] relate the exponent to specific heat and correlation length exponents at the $\mathbf{P}_{\mathrm{O}(m)}$ FP: $\omega_1 = -\alpha/\nu$. The exponent appears to be very small: an adjusted analysis of the 3d six-loop resummed RG expansions results in [8] $\omega_1(m = 2, \forall n) = 0.007(8)$ for the FP $\mathbf{P}_{\mathrm{O}(m)}$, thus providing no definitive answer about its sign. The behaviour of ω_1 in different orders of perturbation theory can be explicitly demonstrated expanding the exponent at $d = 3$ in the pseudo-ε expansion [23] parameter τ up to the six-loop order:

$$\omega_1(m = 2, \forall n) = -1/5\tau + 0.186074\tau^2 - 0.000970\tau^3 + 0.027858\tau^4 - 0.014698\tau^5 + 0.028096\tau^6 \qquad (5)$$

and making an attempt to evaluate the exponent at $\tau = 1$ on the basis of the Padé table [15]:

$$\begin{bmatrix} -0.2000 & -0.0139 & -0.0149 & 0.0130 & -0.0017 & 0.0264 \\ -0.1036 & -0.0149 & -0.0140 & 0.0033 & 0.0079 & o \\ -0.0717 & {\scriptstyle 0.0251} & {\scriptstyle 0.0053} & 0.0209 & o & o \\ -0.0537 & -0.0029 & {\scriptstyle 0.0113} & o & o & o \\ -0.0430 & 0.0630 & o & o & o & o \\ -0.0351 & o & o & o & o & o \end{bmatrix}. \qquad (6)$$

In the table, number of rows and columns corresponds to the orders of denominator and numerator of appropriate Padé approximant for the exponent (5), the small numbers denote unreliable data, obtained on the basis of pole-containing approximants and *o* means that the approximant cannot be constructed. One can see that the table shows no convergence even along the main diagonal and those parallel to it, where the Padé analysis is known to provide the best convergence of results [15]. So these are both the convergence properties of the series (5) and the smallness of the numerical value of $\omega_1(m = 2, \forall n)$ which do not allow us to make a qualitative statement about the stability of the FP $\mathbf{P}_{O(m)}$.

In contrast, if one first defines a pseudo-ε series for the value $m = m_c$ where the exponent $\omega_1(m, \forall n)$ changes its sign, one gets the series which has much better behaviour [32]:

$$m_c = 4 - 8/3\tau + 0.766489\tau^2 - 0.293632\tau^3 + 0.193141\tau^4 - 0.192714\tau^5. \quad (7)$$

Indeed, the corresponding Padé table for m_c reads

4	1.3333	2.0998	1.8062	1.9993	1.8066
2.4	1.9287	1.8875	1.9227	1.9029	*o*
2.0839	1.8799	1.9084	1.9085	*o*	*o*
1.9669	1.9311	1.9085	*o*	*o*	*o*
1.9398	2.2425	*o*	*o*	*o*	*o*
1.9106	*o*	*o*	*o*	*o*	*o*

and leads to the conclusion $m_c < 2$ already in the three-loop order (cf the convergence of the results along the diagonals of the table). A more efficient Padé–Borel–Leroy resummation procedure applied to the series (7) results in an estimate [32] $m_c = 1.912 \pm 0.004$. From here one concludes that $\omega_1(m = 2, \forall n) > 0$, the FP $\mathbf{P}_{O(m)}$ at $m = 2, n = 2$, 3 is stable and governs the critical behaviour of the mn-vector model. In this way, the perturbative RG scheme leads to the results in agreement with general considerations of [12], which show that the theory (1) belongs to the O(2) universality class for these field dimensions.

Moreover, if one repeats the above analysis for the stability exponents at the FP **M**, again by a direct resummation of a series for ω_1 one is led to the numerical estimates, which similar to (6), do not allow for the definite conclusion. In particular, one encounters a controversial situation when both FPs **M** and $\mathbf{P}_{O(2)}$ are simultaneously stable in the same order of perturbation theory. In contrast, being interested in the values of m where the exponent ω_1 at the FP **M** changes its sign, one recovers the series for m_c (7) for any n. In this way, one arrives at a self-consistent picture, where FPs **M** and $\mathbf{P}_{O(m)}$ interchange their stability at $m = m_c$.

Carrying out an analysis of conditions upon which the mn-vector model belongs to the given universality class we encountered two problems which are worth mentioning at the concluding part of this paper. The first is that an analysis of the resummed RG functions directly at fixed space dimensions may lead to an appearance of the unphysical FPs. One of the ways to check the reliability of an analysis is to keep track of the evolution of the given FP with continuous change of d up to the upper critical dimension $d = 4$. The second observation concerns analysis of the FP stability: taking into consideration the contradictory results obtained by a direct analysis of the stability exponents we suggest that the most reliable way to study the boundaries of universality classes in field-theoretical models with several couplings consists of an investigation of the expansions for marginal dimensions. We believe that our observations might be useful in the analysis of critical properties of other field-theoretical models of complicated symmetry.

Acknowledgments

YH and MD were supported in part by the Austrian Fonds zur Förderung der wissenschaftlichen Forschung, project No. 16574-PHY and by the French-Ukrainian cooperation Dnipro project.

The research of TY at the University of Waterloo was undertaken, in part, thanks to funding from the Canada Research Chairs Program (Michel Gingras).

References

[1] Domb C 1996 *The Critical Point* (London: Taylor and Francis) p 376
Chaikin P M and Lubensky T C 1996 *Principles of Condensed Matter Physics* (Cambridge: Cambridge University Press) p 699

[2] Zinn-Justin J 1989 *Quantum Field Theory and Critical Phenomena* (Oxford: Clarendon) p 914
Kleinert H and Schulte-Frohlinde V 2001 *Critical Properties of* ϕ^4*-Theories* (Singapore: World Scientific) p 489

[3] Brézin E, Le Guillou J C and Zinn-Justin J 1974 *Phys. Rev.* B **10** 892

[4] Grinstein G and Luther A 1976 *Phys. Rev.* B **13** 1329

[5] Emery V J 1975 *Phys. Rev.* B **11** 239

[6] Aharony A 1976 *Phase Transitions and Critical Phenomena* vol 6, ed C Domb and M S Green (London: Academic) p 357

[7] Mukamel D 1975 *Phys. Rev. Lett.* **34** 481
Mukamel D and Krinsky S 1975 *J. Phys. C: Solid State Phys.* **8** L496
Bak P and Mukamel D 1976 *Phys. Rep.* B **13** 5078

[8] Pelissetto A and Vicari E 2002 *Phys. Rep.* **368** 549

[9] Carmona J M, Pelissetto A and Vicari E 2000 *Phys. Rev.* B **61** 15136

[10] Folk R, Holovatch Yu and Yavors'kii T 2000 *Phys. Rev.* B **62** 12195
Folk R, Holovatch Yu and Yavors'kii T 2001 *Phys. Rev.* B **63** 189901(E) (erratum)

[11] Folk R, Holovatch Yu and Yavors'kii T 2003 *Sov. Phys.–Usp.* **46** 169 [*Uspekhi Fizicheskikh Nauk* **173** 175] (*Preprint* cond-mat/0106468)

[12] Sak J 1974 *J. Phys. A: Math. Gen.* **8** 1125

[13] De'Bell K and Geldart D 1985 *Phys. Rev.* B **32** 4763
Shpot N A 1988 *Phys. Lett.* A **133** 125
Shpot N A 1989 *Phys. Lett.* A **142** 474
Mudrov A and Varnashev K 1998 *Phys. Rev.* B **57** 3562
Mudrov A and Varnashev K 1998 *Phys. Rev.* B **57** 5704
Sokolov A and Varnashev K 1999 *Phys. Rev.* B **59** 8363
Mudrov A and Varnashev K 2001 *Phys. Rev.* B **64** 214423
Mudrov A and Varnashev K 2001 *JPA* **34** L347

[14] Calabrese P, Orlov E, Pakhnin D and Sokolov A 2004 *Preprint* cond-mat/0405432

[15] Baker G A Jr and Graves-Morris P 1981 *Padé Approximants* (London: Addison-Wesley)

[16] Baker G A, Nickel B G and Meiron D I 1978 *Phys. Rev.* B **17** 1365

[17] 't Hooft G and Veltman M 1972 *Nucl. Phys.* B **44** 189
't Hooft G 1973 *Nucl. Phys.* B **61** 455

[18] Parisi G 1973 *Proc. of the Cargése Summer School* unpublished
Parisi G 1980 *J. Stat. Phys.* **23** 49

[19] Pelissetto A and Vicari E 2000 *Phys. Rev.* B **62** 6393

[20] Kleinert H, Neu J, Schulte-Fohlinde V, Chetyrkin K G and Larin S A 1991 *Phys. Lett.* B **272** 39
Kleinert H, Neu J, Schulte-Fohlinde V, Chetyrkin K G and Larin S A 1993 *Phys. Lett.* B **319** 545 (erratum)

[21] Kleinert H and Schulte-Frohlinde V 1995 *Phys. Lett.* B **342** 284

[22] Wilson K G and Fisher M E 1972 *Phys. Rev. Lett.* **28** 240

[23] Le Guillou J C and Zinn-Justin J 1980 *Phys. Rev.* B **21** 3976

[24] von Ferber C and Holovatch Yu 1997 *Europhys. Lett.* **39** 31
von Ferber C and Holovatch Yu 1997 *Phys. Rev.* E **56** 6370
von Ferber C and Holovatch Yu 1999 *Phys. Rev.* E **59** 6914
von Ferber C and Holovatch Yu 2002 *Phys. Rev.* E **65** 042801
Calabrese P and Parruccini P 2004 *Nucl. Phys.* B **679** 568 (*Preprint* cond-mat/0308037 v2)
Holovatch Yu, Ivaneyko D and Delamotte B 2004 *J. Phys. A: Math. Gen.* **37** 3569 (*Preprint* cond-mat/0312260)

[25] Schloms R and Dohm V 1987 *Europhys. Lett.* **3** 413
Schloms R and Dohm V 1989 *Nucl. Phys.* B **328** 639

[26] Antonenko S A and Sokolov A I 1994 *Phys. Rev.* B **49** 15901

[27] Pelissetto A, Rossi P and Vicari E 2001 *Phys. Rev.* B **63** 140414(R)
[28] Pelissetto A 2003 private communication
[29] Calabrese P, Parruccini P, Pelissetto A and Vicari E 2004 *Preprint* cond-mat/0405667
[30] Dudka M, Holovatch Yu and Yavorskii T 2002 *Acta Phys. Slovaca* **52** 323
[31] Bervillier C 1986 *Phys. Rev.* B **34** 8141
[32] Dudka M, Holovatch Yu and Yavors'kii T 2001 *J. Phys. Stud.* **5** 233